

NAG Toolbox for MATLAB

s21ca

1 Purpose

s21ca evaluates the Jacobian elliptic functions sn, cn and dn.

2 Syntax

```
[sn, cn, dn, ifail] = s21ca(u, m)
```

3 Description

s21ca evaluates the Jacobian elliptic functions of argument u and parameter m ,

$$\begin{aligned}\operatorname{sn}(u \mid m) &= \sin \phi, \\ \operatorname{cn}(u \mid m) &= \cos \phi, \\ \operatorname{dn}(u \mid m) &= \sqrt{1 - m \sin^2 \phi},\end{aligned}$$

where ϕ , called the *amplitude* of u , is defined by the integral

$$u = \int_0^\phi \frac{d\theta}{\sqrt{1 - m \sin^2 \theta}}.$$

The elliptic functions are sometimes written simply as $\operatorname{sn} u$, $\operatorname{cn} u$ and $\operatorname{dn} u$, avoiding explicit reference to the parameter m .

Another nine elliptic functions may be computed via the formulae

$$\begin{aligned}\operatorname{cd} u &= \operatorname{cn} u / \operatorname{dn} u \\ \operatorname{sd} u &= \operatorname{sn} u / \operatorname{dn} u \\ \operatorname{nd} u &= 1 / \operatorname{dn} u \\ \operatorname{dc} u &= \operatorname{dn} u / \operatorname{cn} u \\ \operatorname{nc} u &= 1 / \operatorname{cn} u \\ \operatorname{sc} u &= \operatorname{sn} u / \operatorname{cn} u \\ \operatorname{ns} u &= 1 / \operatorname{sn} u \\ \operatorname{ds} u &= \operatorname{dn} u / \operatorname{sn} u \\ \operatorname{cs} u &= \operatorname{cn} u / \operatorname{sn} u\end{aligned}$$

(see Abramowitz and Stegun 1972).

s21ca is based on a procedure given by Bulirsch 1960, and uses the process of the arithmetic-geometric mean (16.9 in Abramowitz and Stegun 1972). Constraints are placed on the values of u and m in order to avoid the possibility of machine overflow.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Bulirsch R 1960 Numerical calculation of elliptic integrals and elliptic functions *Numer. Math.* **7** 76–90

5 Parameters

5.1 Compulsory Input Parameters

- 1: **u** – double scalar
- 2: **m** – double scalar

The argument u and the parameter m of the functions, respectively.

Constraints:

$$\begin{aligned} \text{abs}(\mathbf{u}) &\leq \sqrt{\lambda}, \text{ where } \lambda = 1/\text{x02am}; \\ \text{if } \text{abs}(\mathbf{u}) < 1/\sqrt{\lambda}, \text{ abs}(\mathbf{m}) &\leq \sqrt{\lambda}. \end{aligned}$$

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

- 1: **sn** – double scalar
- 2: **cn** – double scalar
- 3: **dn** – double scalar

The values of the functions $\text{sn } u$, $\text{cn } u$ and $\text{dn } u$, respectively.

- 4: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, $\text{abs}(\mathbf{u}) > \sqrt{\lambda}$, where $\lambda = 1/\text{x02am}()$.

ifail = 2

On entry, $\text{abs}(\mathbf{m}) > \sqrt{\lambda}$ and $\text{abs}(\mathbf{u}) < 1/\sqrt{\lambda}$.

7 Accuracy

In principle the function is capable of achieving full relative precision in the computed values. However, the accuracy obtainable in practice depends on the accuracy of the standard elementary functions such as SIN and COS.

8 Further Comments

None.

9 Example

```
u = 0.2;
```

```
m = 0.3;  
[sn, cn, dn, ifail] = s21ca(u, m)
```

```
sn =  
    0.1983  
cn =  
    0.9801  
dn =  
    0.9941  
ifail =  
        0
```
